Calculus Warm Up #3-5

Find the net area and the total area between the curve and the x-axis. (Draw the picture first!)

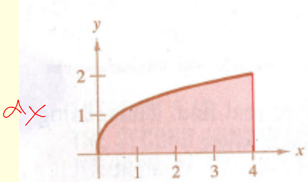
$y = 4 - x^2$ on $[0, 3]$

HW Questions, p. 267

In Exercises 25–30, determine the area of the indicated region.

27) $y = 1 - x^4$

29. $y = \sqrt[3]{2x}$



29) $\int_0^4 \sqrt[3]{2}\,\sqrt[3]{x}\,dx$

$\sqrt[3]{2}\int_0^4 x^{1/3}\,dx$

$\sqrt[3]{2}\left[\frac{3x^{4/3}}{4}\right]_0^4$

$\sqrt[3]{2}\left(\frac{3(4)^{4/3}}{4} - 0\right)$

$2^{1/3} \cdot 3(4)^{1/3}$

$8^{1/3} \cdot 3$

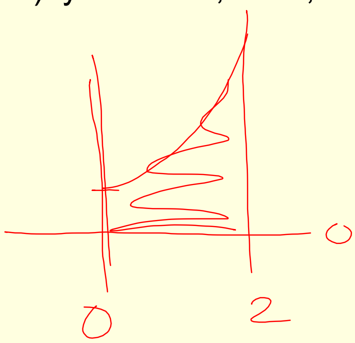
$2 \cdot 3$

6

In Exercises 31–34, find the area of the region bounded by the graphs of the given equations.

31) $y = 3x^2 + 1$, $x = 0$, $x = 2$, $y = 0$

33. $y = x^3 + x$, $x = 2$, $y = 0$



In Exercises 35–38, find the values of c guaranteed by the *Mean Value Theorem for Integrals* for the given function over the specified interval.

35) $f(x) = x^3$ $[0, 2]$

	Function	*Interval*
37.	$f(x) = -x^2 + 4x$	$[0, 3]$

In Exercises 39–42, sketch the graph of the given function over the specified interval. Find the average value of the function over the interval and all values of x where the function equals its average value.

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Function	Interval
39) $f(x) = 4 - x^2$	$[-2, 2]$

$$f(c) = \frac{1}{4}\int_{-2}^{2} (4 - x^2)\,dx$$

$$f(c) = \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$f(c) = \frac{8}{3}$$

$$f(c) = 4 - c^2$$

$$4 - c^2 = \frac{8}{3}$$

In Exercises 43–48, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result of part (a).

43) $F(x) = \int_0^x (t+2)\,dt$

45. $F(x) = \int_8^x \sqrt[3]{t}\,dt$

47) $F(x) = \int_1^x \frac{1}{t^2}\,dt$

In Exercises 49–52, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

49. $F(x) = \int_{-2}^{x} (t^2 - 2t + 5)\, dt$

51) $F(x) = \int_{-1}^{x} \sqrt{t^4 + 1}\, dt$

$F'(x) = \sqrt{x^4 + 1}$

$$\frac{d}{dx} \int F(x)\, dx = f(x)$$

5.5 Pattern Recognition

Change of Variables

U-Substitution

The General Power Rule for Integration

Evaluating Integrals using Pattern Recognition

$\int (x^2 - \sin x)\, dx$

$\frac{x^3}{3} + \cos x + C$

$\int \frac{1}{x}\, dx =$

$\ln|x| + C$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$

Changing the Variable:

Helpful when we don't recognize the integrand

<u>Exploration</u>

Let $f(x) = x^3 + 1$ and $u = x^2$

find:

$\int f(x)\,dx$

$\frac{x^4}{4} + x + C$

$\int f(u)\,du$

$\int (u^3+1)\,du$

$\frac{u^4}{4} + u + C$

$\frac{x^8}{4} + x^2 + C$

Don't match

$\int f(u)\,dx$

$\int (u^3+1)\,dx$

$\int (x^6+1)\,dx$

$\frac{x^7}{7} + x + C$

Using change in variable:

"U - Substituion"

$$\int x^2\sqrt{5+2x^3}\,dx$$

Let $u = 5+2x^3$

$\cancel{dx}\cdot\frac{du}{\cancel{dx}} = \underbrace{6x^2\,dx}$

$$\frac{1}{6}\int \underbrace{\sqrt{5+2x^3}}\cdot\underbrace{6x^2\,dx}$$

$$\frac{1}{6}\int u^{1/2}\,du$$

$$\frac{1}{6}\cdot\frac{2u^{3/2}}{3}+C$$

$$\frac{(5+2x^3)^{3/2}}{9}+C$$

You try:

$$\int x(x^2+1)^2\,dx$$

Let u = x^2+1

du = $2x\,dx$

$$=\frac{1}{2}\int \underbrace{(2x\,dx)}_{du}\underbrace{(x^2+1)}_{u}{}^2$$

$$=\frac{1}{2}\int u^2\,du$$

$$\frac{1}{2}\left[\frac{u^3}{3}\right]+C$$

$$\frac{1}{2}\left[\frac{u^3}{3}+C\right]$$

$$\frac{(x^2+1)^3}{6}+C$$

Tricky...

$$\int \frac{30x^2}{(5x^3-2)^2}\,dx$$

let u = $5x^3 - 2$

du = $15x^2\,dx$

$$2\int 15x^2(5x^3-2)^{-2}\,dx$$

$$2\int u^{-2}\,du$$

$$2\left[\frac{u^{-1}}{-1}\right] + C$$

$$-\frac{2}{5x^3-2} + C$$

HW: p. 278, # 1 - 4,

5 - 27 odd

In Exercises 1–4, complete the table by identifying u and du for the given integral.

p. 278

	u	du
1. $\int (5x^2 + 1)^2(10x)\, dx$	$5x^2+1$	$10x\, dx$
2. $\frac{1}{3}\int 3x^2\sqrt{x^3+1}\, dx$	x^3+1	$3x^2\, dx$
3. $\frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}}\, dx$	x^2+1	$2x\, dx$
4. $\int (x^3+3)(3x^2)\,dx$	x^3+3	$3x^2 dx$